A– LEVEL TOPIC REVIEW

unit C2 **factor and remainder theorems**

---

1. Use the Factor Theorem to show that:
   a) $(x-1)$ is a factor of $x^3-2x^2-11x+12$. **(2 marks)**
   b) $(x+3)$ is a factor of $-x^3-x^2+7x+3$. **(2 marks)**
   c) $(2x-1)$ is a factor of $2x^3-x^2+2x-1$. **(2 marks)**

2. Use algebraic division to find the quotient and remainder (if any) when :
   a) x^3+2x^2+3x+2 is divided by $(x+1)$. **(3 marks)**
   b) $2x^3+4x^2-5x-3$ is divided by $(x+2)$. **(3 marks)**
   c) x^3+2x^2-4 is divided by $(x-3)$. **(3 marks)**

3. In the following equations one solution is given. Factorise each expression and hence solve the equation:
   a) $x^3-6x^2+11x-6=0$; $x=1$ is one solution. **(3 marks)**
   b) $2x^3-x^2-5x-2=0$; $x=-\frac{1}{2}$ is one solution. **(3 marks)**

4. If $(x+2)$ is a factor of x^3-2x^2+ax+6, find the value of *a*. **(3 marks)**

5. Find the remainder when :
   a) x^2+5x-2 is divided by $(x-2)$. **(2 marks)**
   b) $2x^3+x^2-5x+1$ is divided by $(2x-1)$. **(3 marks)**

6. $\mathrm{f}(x)=2x^3+5x^2-7x-3$. Show that $(x-2)$ is not a factor of $\mathrm{f}(x)$, and find the remainder when $\mathrm{f}(x)$ is divided by $(2x+3)$. **(5 marks)**

7. The expression $\mathrm{f}(x)=x^3+ax^2+bx+10$ is divisible by $(x+2)$ but leaves a remainder of 12 when divided by $(x+1)$
   a) Find the values of the constants *a* and *b*. **(6 marks)**
   b) Solve the equation $\mathrm{f}(x)=0$. **(4 marks)**

8. When x^3+ax^2+2x+1 is divided by $(x-2)$ the remainder is three times the remainder when divided by $(x-1)$. Find the value of *a*. **(6 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **factor and remainder theorems**

---

1. a) $f(1) = 1 - 2 - 11 + 12 = 0$ M1A1

   b) $f(-3) = 27 - 9 - 21 + 3 = 0$ M1A1

   c) $f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} + 1 - 1 = 0$ M1A1

2. a) quotient : $x^2 + x + 2$ M1A1

   remainder : 0 A1

   b) quotient : $2x^2 - 5$ M1A1

   remainder : 7 A1

   c) quotient : $x^2 + 5x + 15$ M1A1

   remainder : 41 A1

3. a) $(x-1)(x^2 - 5x + 6) = 0$ M1

   $(x-2)(x-3) = 0$ M1

   $x = 2,\ x = 3$ A1

   b) $(2x+1)(x^2 - x - 2) = 0$ M1

   $(x+1)(x-2) = 0$ M1

   $x = -1,\ x = 2$ A1

4. $f(-2) = 0$ M1

   $-8 - 8 - 2a + 6 = 0$ M1

   $a = -5$ A1

5. a) $f(2) = 4 + 10 - 2 = 12$ M1A1

   b) $f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 1 = -1$ M2A1

6. $f(2) = 16 + 20 - 14 - 3 = 19 \neq 0$ M1A1

   $f\left(-\frac{3}{2}\right) = -\frac{27}{4} + \frac{45}{4} + \frac{21}{2} - 3 = 12$ M2A1

7. a) $f(-2) = 0$ M1

   $-8 + 4a - 2b + 10 = 0$ A1

   $f(-1) = 12$ M1

   $-1 + a - b + 10 = 12$ A1

   $a = -4,\ b = -7$ M1A1

   b) $(x+2)(x^2 - 6x + 5) = 0$ M1

   $(x+2)(x-1)(x-5) = 0$ M1

   $x = -2,\ x = 1,\ x = 5$ A2

8. $f(2) = 8 + 4a + 4 + 1 = 4a + 13$ M1A1

   $f(1) = 1 + a + 2 + 1 = a + 4$ M1A1

   $4a + 13 = 3(a + 4)$ M1

   $a = -1$ A1

A– LEVEL TOPIC REVIEW

unit C2 coordinate geometry of the circle

---

1. Write down the centres and radii of the following circles.

   a) $x^2 + y^2 = 4$ b) $(x-3)^2 + y^2 = 49$ c) $(x+3)^2 + (y-5)^2 = 1.96$ **(6 marks)**

2. Find the equations of the following circles with the following properties.

   a) centre (3, 4), radius 6. **(2 marks)**

   b) centre (5, –2), radius $\frac{1}{2}$. **(2 marks)**

   c) centre (4, 3), passing through the point (1, 1). **(3 marks)**

   d) the points (2, 8) and (–1, –1) are opposite ends of the diameter. **(3 marks)**

3. a) Show that the point *P* (3, 5) lies on the circle $(x-4)^2 + (y-1)^2 = 17$. **(1 mark)**

   b) Find the equation of the tangent to the circle at the *P*. giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **(4 marks)**

4. The circle $(x-2)^2 + (y-3)^2 = 4$ has centre *C* and radius *r*.

   a) Write down the coordinates of *C* and the value of *r*. **(2 marks)**

   A tangent to the circle passes through the point *P* (5, 7) and touches the circle at *T*

   b) Draw a sketch of the circle showing clearly the positions of *P*, *T* and *C*. **(2 marks)**

   c) Hence calculate the length *PT*. **(3 marks)**

5. The points *A* (1, 5), *B* (7, 9) and *C* (2, 4) lie on a circle.

   a) Find the gradients of *AC* and *BC*. **(2 marks)**

   b) Explain what your answer to a) tells you about the line *AB*. **(1 mark)**

   c) Find the equation of the circle passing through *A*, *B* and *C*. **(4 marks)**

6. a) Find the coordinates of the two points *A* and *B* where the line $y = 2x - 4$ intersects with the circle $(x-1)^2 + y^2 = 25$. **(4 marks)**

   b) Sketch the circle and the line showing clearly the position of *A* and *B*. **(2 marks)**

   c) *M* is the mid point of *AB*. Write down the equation of the line passing through *M* and the centre of the circle. **(4 marks)**

7. A circle has equation $(x-1)^2 + (y+3)^2 = 16$. Find the distance between the point *P* (6, 9) and the nearest point on the circle to *P*. **(5 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **coordinate geometry of the circle**

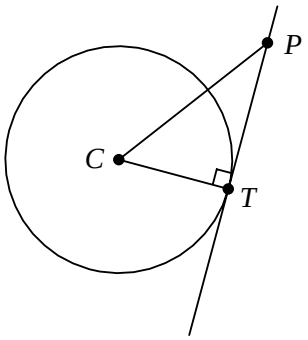
---

1. a) (0, 0); 2 M1A1
   b) (3, 0); 7 M1A1
   c) (-3, 5); 1.4 M1A1

2. a) $(x-3)^2+(y-4)^2=36$ M1A1
   b) $(x-5)^2+(y+2)^2=\frac{1}{4}$ M1A1
   c) $r^2=(4-1)^2+(3-1)^2=13$ M1A1
   $(x-4)^2+(y-3)^2=13$ A1
   d) centre (0.5, 3.5) B1
   $r^2=(2-0.5)^2+(8-3.5)^2=22.5$ M1
   $(x-0.5)^2+(y-3.5)^2=22.5$ A1

3. a) $(3-4)^2+(5-1)^2=17$ B1
   b) gradient of radius $=\dfrac{-4}{1}=-4$ M1A1
   $y-5=\frac{1}{4}(x-3)$ M1
   $x-4y+17=0$ A1

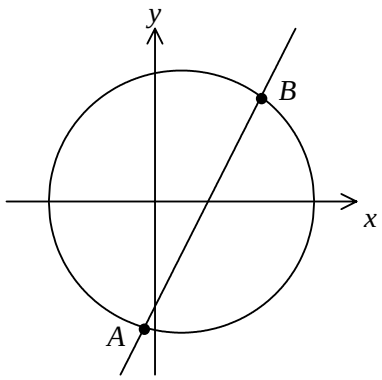
4. a) (2, 3); 2 M1A1
   b) G2



   c) $CP=\sqrt{3^2+4^2}=5$ A1
   $PT=\sqrt{5^2-2^2}=\sqrt{21}$ M1A1

5. a) $AC:\dfrac{4-5}{2-1}=-1$ M1
   $BC:\dfrac{4-9}{2-7}=1$ A1
   b) C is a right angle, and so AB must be a diameter (‘angle in a semicircle’) A1
   c) centre (4, 7) A1
   $r^2=(7-4)^2+(9-7)^2=13$ M1A1
   $(x-4)^2+(y-7)^2=13$ A1

6. a) $(x-1)^2+(2x-4)^2=25$ M1
   $5x^2-18x-8=0$
   $(x-4)(5x+2)=0$ A1
   (4, 4); (−0.4, −4.8) M1A1
   b) G2



   c) M (1.8, −0.4) B1
   centre (1, 0) B1
   gradient $=\dfrac{-0.4}{0.8}=-\frac{1}{2}$ M1
   $y-0=-\frac{1}{2}(x-1)$
   $y=-\frac{1}{2}x+\frac{1}{2}$ A1

7. radius = 4 A1
   centre C (1, −3) A1
   $CP=\sqrt{(6-1)^2+(9--3)^2}=13$ M1
   distance $=13-4=9$ M1A1

A– LEVEL TOPIC REVIEW

unit C2 **geometric series**

---

1. A geometric series has a first term of 3 and a common ratio of 2.
   a) Write down the first four terms of the series and the n^{th} term. **(3 marks)**
   b) Calculate the sum of the first 20 terms. **(2 marks)**

2. The 5^{th} term of a geometric sequence is 405 and the 9^{th} term is 32805. All the terms are positive.
   a) Find the common ratio. **(4 marks)**
   b) Find the first term. **(2 marks)**
   c) Find the sum of the first 10 terms. **(2 marks)**

3. The first three terms of a geometric progression are $2x-6$, $6-3x$ and $5x-2$. Find two possible values of x and the corresponding common ratios of the sequence. **(8 marks)**

4. $1+\frac{3}{2}+\frac{9}{4}+...+\frac{x}{512}$ is a geometric series
   a) Find the value of x. **(3 marks)**
   b) Find the number of terms of the series. **(1 mark)**
   c) Find the sum of the series. **(2 marks)**

5. Evaluate $\sum_{r=1}^{\infty}\frac{4}{3^r}$ **(4 marks)**

6. Find the sum of the following infinite geometric series.
   a) $4+2+1+...$ **(2 marks)**
   b) $1-\frac{1}{\sqrt{2}}+\frac{1}{2}-\frac{1}{2\sqrt{2}}+...$, leaving your answer in the simplest surd form. **(5 marks)**

7. A man invests £100 in a savings account on January 1^{st} every year, starting in 2000. The account pays 5% interest on the 31^{st} December each year.
   a) How much money does he have in his account
      (i) On 31^{st} December 2000. **(1 marks)**
      (ii) On 31^{st} December 2001. **(2 marks)**
   b) Write down a geometric series, the sum of which gives the amount of money in his account on 31^{st} December 2020. Find the sum of this series. **(4 marks)**
   c) After how many years will the account first exceed £25000? **(5 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

1. a) 3, 6, 12, 24 A1

$3\times 2^{n-1}$ M1A1

b) $\dfrac{3\left(2^{20}-1\right)}{2-1}=3145725$ M1A1

2. a) $ar^4=405$ M1

$ar^8=32805$ A1

$r^4=\dfrac{32805}{405}=81\Rightarrow r=3$ M1A1

b) $a\times 3^4=405\Rightarrow a=5$ M1A1

c) $\dfrac{5\left(3^{10}-1\right)}{3-1}=147620$ M1A1

3. $r=\dfrac{6-3x}{2x-6}=\dfrac{5x-2}{6-3x}$ M1

$(6-3x)^2=(5x-2)(2x-6)$ M1

$9x^2-36x+36=10x^2-34x+12$ A1

$x^2+2x-24=0$

$(x+6)(x-4)=0$ M1

$x=-6\Rightarrow r=\dfrac{24}{-18}=-\frac{4}{3}$ M1A1

$x=4\Rightarrow r=\dfrac{-6}{2}=-3$ M1A1

4. a) $512=2^9$ M1

$x=3^9=19683$ M1A1

b) 10 terms M1

c) $\dfrac{1\left(1.5^{10}-1\right)}{1.5-1}=113.330...$ M1A1

5. $a=\frac{4}{3}$, $r=\frac{1}{3}$ M1A1

$\dfrac{\frac{4}{3}}{1-\frac{1}{3}}=2$ M1A1

6. a) $a=4$, $r=\frac{1}{2}$ A1

$\dfrac{4}{1-\frac{1}{2}}=8$ A1

b) $a=1$, $r=-\dfrac{1}{\sqrt{2}}$ A1

$\dfrac{1}{1-\dfrac{1}{\sqrt{2}}}=\dfrac{\sqrt{2}}{\sqrt{2}-1}$ M1A1

$\dfrac{\sqrt{2}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$ M1

$2+\sqrt{2}$ A1

7. a) (i) $100\times 1.05=£105$ A1

(ii) $105+105\times 1.05=£215.25$ M1A1

b) $105+$
$105\times 1.05+$
$105\times 1.05^2+...+$
105×1.05^{20} M1A1

$\dfrac{105\left(1.05^{21}-1\right)}{1.05-1}=£3750.52$ M1A1

c) $\dfrac{105\left(1.05^n-1\right)}{1.05-1}>25000$ M1

$105\left(1.05^n-1\right)>1250$

$1.05^n-1>11.904...$ A1

$n\log 1.05>\log 12.904$ M1

$n>52.43...$ A1

$n=53$ A1

(or trial and improvement M2A1)

A– LEVEL TOPIC REVIEW

unit C2 **binomial theorem**

---

1. Use the binomial theorem to expand:
   a) $(2x+5)^4$ **(3 marks)**
   b) $(3x-7)^5$ **(4 marks)**
   c) $\left(2x-\frac{3}{x}\right)^3$ **(3 marks)**

2. a) Expand $(a+b)^4$. **(2 marks)**
   b) Hence write down the expansion of $(a-b)^4$. **(1 mark)**
   c) Hence simplify $\left(\sqrt{2}+\sqrt{3}\right)^4-\left(\sqrt{2}-\sqrt{3}\right)^4$, giving your answer in the form $a\sqrt{b}$. **(4 marks)**

3. a) Expand $(1-2x)^9$ in ascending powers of x up to and including the term in x^3. **(2 marks)**
   b) Use your expansion to find an approximation to 0.98^9, correct to 4 d.p. **(3 marks)**

4. Show that the first three terms of the expansion of $(1+x)^7(1+2x)^4$ are $1+15x+101x^2$.
   (7 marks)

5. When $(1+ax)^n$ is expanded, the first three terms are $1+2x+\frac{15}{8}x^2$. Find the values of a and n.
   (8 marks)

6. Find the coefficient of the term in x^8 in the expansion of $(7x^2+3x)^6$. **(5 marks)**

7. a) Expand $(2+x)^5$. **(2 marks)**
   b) Hence write down the expansion of $(2-x)^5$. **(1 mark)**
   c) Hence solve the equation $(2+x)^5+(2-x)^5=464$. **(5 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **binomial theorem**

---

1. a) $16x^4+160x^3+600x^2+1000x+625$ M2A1

1	4	6	4	1
$16x^4$	$8x^3$	$4x^2$	$2x$	
	5	25	125	625

b) $243x^5-2835x^4+13230x^3$ M3A1
$-30870x^2+36015x-16807$

1	5	10	10	5	1
$243x^5$	$81x^4$	$27x^3$	$9x^2$	$3x$	
	-7	49	-343	2401	-16807

c) $8x^3-36x+\frac{54}{x}-\frac{27}{x^3}$ M2A1

1	3	3	1
$8x^3$	$4x^2$	$2x$	
	$-\frac{3}{x}$	$\frac{9}{x^2}$	$-\frac{27}{x^3}$

2. a) $a^4+4a^3b+6a^2b^2+4ab^3+b^4$ M1A1

b) $a^4-4a^3b+6a^2b^2-4ab^3+b^4$ B1

c) $8a^3b+8ab^3$ M1

$8\times2\sqrt{2}\times\sqrt{3}+8\times\sqrt{2}\times3\sqrt{3}$ M1A1

$40\sqrt{6}$ A1

3. a) $1-18x+144x^2-672x^3$ M1A1

1	9	36	84
	$-2x$	$4x^2$	$-8x^3$

b) $x=0.01$ B1

$1-0.18+0.0144-0.000672$ M1

0.8337 A1

4. $(1+x)^7=1+7x+21x^2$ M1A1

1	7	21
	x	x^2

$(1+2x)^4=1+8x+24x^2$ M1A1

1	4	6
	$2x$	$4x^2$

$(1+x)^7(1+2x)^4=1+15x+101x^2$ M2A1

	1	$7x$	$21x^2$
1	1	$7x$	$21x^2$
$8x$	$8x$	$56x^2$	–
$24x^2$	$24x^2$	–	–

5. $na=2$ A1

$\frac{n(n-1)}{2}a^2=\frac{15}{8}$ M1A1

$\frac{n(n-1)}{2}\times\frac{4}{n^2}=\frac{15}{8}$ M1A1

$16(n-1)=15n$ A1

$n=16$ A1

$a=\frac{1}{8}$ A1

6. $\binom{6}{4}\left(7x^2\right)^2(3x)^4$ M2A1

$15\times49\times81=59535$ M1A1

7. a) $x^5+10x^4+40x^3+80x^2+80x+32$ M1A1

1	5	10	10	5	1
	2	4	8	16	32
x^5	x^4	x^3	x^2	x	

b) $-x^5+10x^4-40x^3+80x^2-80x+32$ B1

c) $20x^4+160x^2+64=464$ M1

$20x^4+160x^2-400=0$ A1

$x^4+8x^2-20=0$ M1

$5\left(x^2-2\right)\left(x^2+10\right)=0$ A1

$x=\pm\sqrt{2}$ A1

A– LEVEL TOPIC REVIEW

unit C2 **trigonometry**

---

1. Convert the following angles, which are given in radians, to degrees:

   a) π b) $\frac{1}{4}\pi$ c) $\frac{1}{3}\pi$ d) -2π **(4 marks)**

2. Express the following angles in radians, giving each answer in terms of π:

   a) $90°$ b) $360°$ c) $30°$ d) $540°$ **(4 marks)**

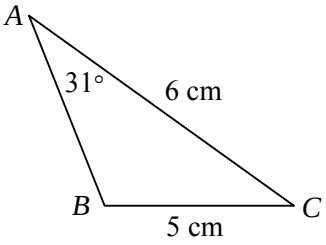
3. A sector *AOB* is formed from a circle, centre *O*, radius 5cm where angle $AOB = \frac{5}{6}\pi$.

   a) Calculate the length of the arc. **(2 marks)**

   b) If the chord *AB* is drawn, calculate the area of the segment formed. **(5 marks)**

4. a) Given that *B* is obtuse, find the missing lengths and angles of this triangle. **(6 marks)**

   b) Calculate the area of the triangle. **(2 marks)**



5. Solve the following equations for values within the given range:

   a) $\sin x = 0.5, \quad -180° \le x \le 180°$ **(2 marks)**

   b) $2\cos x = \sqrt{2}, \quad 0 \le x \le 2\pi$ **(2 marks)**

   c) $\tan(x + 50) = 1, \quad 0° \le x \le 540°$ **(3 marks)**

   d) $\cos\left(2x + \frac{1}{3}\pi\right) = -\frac{1}{2}, \quad -\frac{1}{2}\pi \le x \le \frac{1}{2}\pi$ **(3 marks)**

6. Solve the following equations for $0° \le x \le 360°$, giving your answers correct to the nearest degree:

   a) $2\sin^2 x - \sin x = 0$ **(3 marks)**

   b) $1 - \cos x - 2\sin^2 x = 0$ **(5 marks)**

   c) $1 + \tan x = \dfrac{2}{\tan x}$ **(4 marks)**

7. Prove the following identities:

   a) $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x \cos x}$ **(3 marks)**

   b) $(\sin x + \cos x)^2 - 1 = 2\sin x \cos x$ **(2 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **trigonometry**

---

1. a) $180°$ B1
   b) $45°$ B1
   c) $60°$ B1
   d) $-360°$ B1

2. a) $\frac{1}{2}\pi$ B1
   b) 2π B1
   c) $\frac{1}{6}\pi$ B1
   d) 3π A1

3. a) $5\times\frac{5}{6}\pi = 13.09\,\text{cm}$ M1A1
   b) $\text{sector} = \frac{1}{2}\times 5^2\times\frac{5}{6}\pi = \frac{125}{12}\pi$ M1A1
   $\text{triangle} = \frac{1}{2}\times 5^2\sin\frac{5}{6}\pi = \frac{25}{4}$ M1A1
   $\text{segment} = \frac{125}{12}\pi - \frac{25}{4} = 26.47\ \text{cm}^2$ A1

4. a) $\dfrac{\sin B}{6} = \dfrac{\sin 31°}{5}$ M1
   $B = 141.8°$ A1
   $C = 180 - 31 - 141.8 = 7.2°$ M1A1
   $c^2 = 5^2 + 6^2 - 2\times 5\times 6\times\cos 7.2°$ M1
   $c = 1.21\ \text{cm}$ A1
   b) $\frac{1}{2}\times 5\times 6\times\sin 7.2° = 1.87\ \text{cm}^2$ M1A1

5. a) $x = 30°$ M1
   $x = 180 - 30 = 150°$ A1
   b) $x = \frac{1}{4}\pi$ M1
   $x = 2\pi - \frac{1}{4}\pi = \frac{7}{4}\pi$ A1
   c) $x + 50 = \cancel{45}, 225, 405, 585$ M1A1
   $x = 175°,\ 355°,\ 535°$ A1
   d) $2x + \frac{1}{3}\pi = -\frac{2}{3}\pi,\ \frac{2}{3}\pi,\ \frac{4}{3}\pi$ M1A1
   $x = -\frac{1}{2}\pi,\ x = \frac{1}{6}\pi,\ x = \frac{1}{2}\pi$ A1

6. a) $\sin x(2\sin x - 1) = 0$ M1
   $\sin x = 0 \Rightarrow x = 0°, 180°, 360°$ A1
   $\sin x = \frac{1}{2} \Rightarrow x = 30°, 150°$ A1
   b) $1 - \cos x - 2\left(1 - \cos^2 x\right) = 0$ M1
   $2\cos^2 x - \cos x - 1 = 0$ A1
   $(2\cos x + 1)(\cos x - 1) = 0$ M1
   $\cos x = -\frac{1}{2} \Rightarrow x = 120°,\ 240°$ A1
   $\cos x = 1 \Rightarrow x = 0°,\ 360°$ A1
   c) $\tan x + \tan^2 x = 2$ M1
   $\tan^2 x + \tan x - 2 = 0$
   $(\tan x + 2)(\tan x - 1) = 0$ M1
   $\tan x = -2 \Rightarrow x = 117°,\ 297°$ A1
   $\tan x = 1 \Rightarrow x = 45°,\ 225°$ A1

7. a) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$ M1
   $\dfrac{\sin^2 x + \cos^2 x}{\sin x\cos x}$ M1
   $\dfrac{1}{\sin x\cos x}$ A1
   b) $\sin^2 x + 2\sin x\cos x + \cos^2 x - 1$ M1
   $2\sin x\cos x$ A1

A– LEVEL TOPIC REVIEW

unit C2 **exponentials and logarithms**

---

1. Sketch, on the same set of axes, the graphs of :

   a) $y = 1^x$ b) $y = 2^x$ c) $y = 3^x$ d) $y = \left(\frac{1}{2}\right)^x$ **(4 marks)**

2. Evaluate:

   a) $\log_5\left(\frac{1}{125}\right)$ b) $\log_4 2$ c) $\log_4 32$

   d) $\log_a a\sqrt{a}$ e) $\dfrac{\log_a 125}{\log_a 5}$ **(11 marks)**

3. Express as a single logarithm:

   a) $\log_a 2 + \log_a 3$ b) $2\log_a 5 - \log_a 10$

   c) $\log_a 2 + 3\log_a 3 - 2\log_a 6$ d) $\log_a 6 + 2\log_a 3 + 1$ **(7 marks)**

4. Express in terms of $\log a$, $\log b$ and $\log c$:

   a) $\log\left(\dfrac{ab^2}{c}\right)$ b) $\log\left(\dfrac{a^2\sqrt{b}}{\sqrt[3]{c}}\right)$ **(4 marks)**

5. Solve the following equations:

   a) $4^{2x-5} = 64$ b) $2^x = 10$

   c) $4^x - 10\left(2^x\right) + 16 = 0$ d) $7^{x-3} = 4^{2x}$ **(12 marks)**

6. Solve the following equations:

   a) $\log_{10} 2 + \log_{10} x = 3$ b) $\log_{10}\left(n^2 - 90n\right) = 3$ **(5 marks)**

7. a) Explain why $2 < \log_7 60 < 3$. **(1 marks)**

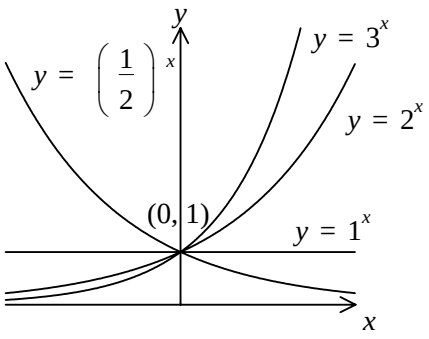
   b) Find the value of $\log_7 60$, giving your answer to three decimal places. **(3 marks)**

8. Find $\sum_{r=1}^{n} \log_{10}(a^r)$, giving your answer in terms of a and n. **(3 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **exponentials and logarithms**

---

1. 

G1 G1 G1 G1

2. a) $5^x = \frac{1}{125}$ M1
   $x = -3$ A1
   b) $4^x = 2$ M1
   $x = \frac{1}{2}$ A1
   c) $4^x = 32$ M1
   $2^{2x} = 32 \Rightarrow x = 2.5$ M1A1
   d) $\log_a a^{\frac{3}{2}} = \frac{3}{2}$ M1A1
   e) $\log_a 5^3 = 3\log_a 5$ M1
   $x = 3$ A1

3. a) $\log_a(2\times 3) = \log_a 6$ A1
   b) $\log_a \frac{5^2}{10} = \log_a 2.5$ M1A1
   c) $\log_a \frac{2\times 3^3}{6^2} = \log_a 1.5$ M1A1
   d) $\log_a(6\times 3^2 \times a) = \log_a 54a$ M1A1

4. a) $\log a + 2\log b - \log c$ M1A1
   b) $2\log a + \frac{1}{2}\log b - \frac{1}{3}\log c$ M1A1

5. a) $4^{2x-5} = 4^3$ M1
   $2x - 5 = 3 \Rightarrow x = 4$ A1
   b) $x\log 2 = \log 10$ M1
   $x = 3.322$ A1
   c) $2^{2x} - 10(2^x) + 16 = 0$ M1
   $(2^x - 2)(2^x - 8) = 0$ A1
   $x = 1,\ x = 3$ M1A1
   d) $(x-3)\log 7 = 2x\log 4$ M1
   $x\log 7 - 2x\log 4 = 3\log 7$ A1
   $x = \frac{3\log 7}{\log 7 - 2\log 4} = -7.062$ M1A1

6. a) $\log_{10} 2x = 3$ M1
   $2x = 10^3 \Rightarrow x = 500$ A1
   b) $n^2 - 90n = 1000$ M1
   $(n+10)(n-100) = 0$ M1
   $n = -10,\ n = 100$ A1

7. a) $7^2 < 60 < 7^3$ B1
   b) $7^x = 60$ M1
   $x\log 7 = \log 60$ M1
   $x = 2.104$ A1

8. $\log_{10} a\sum_{r=1}^{n} r$ M1
   $\frac{1}{2}n(n+1)\log_{10} a$ M1A1

A– LEVEL TOPIC REVIEW

unit C2 **differentiation**

---

1. Use differentiation to find the values of x for which the function $\mathrm{f}(x) = x^3 - 6x^2 + 9x - 2$ is an increasing function. **(4 marks)**

2. Use differentiation to find the coordinates (as fractions, not decimals!) and natures of the turning points of the following curves.

   a) $y = \dfrac{3 - x^2}{x^3}$ b) $y = x^5 - 15x^3$ **(17 marks)**

3. $\mathrm{f}(x) = 3x^2 - x^3$.

   a) Find $\mathrm{f}'(x)$. **(1 mark)**

   b) Find the coordinates of the stationary points, and determine their natures. **(6 marks)**

   c) Find the range of values for which the function is decreasing. **(2 marks)**

   d) Sketch the curve $y = \mathrm{f}(x)$ marking clearly the coordinates of any turning points and intercepts with the axes. **(3 marks)**

4. The height, h metres, of a ball above ground level is given by the formula $h = 2 + 9t - 5t^2$, where t is the time elapse in seconds.

   a) Find the height of the ball when $t = 0$. **(1 mark)**

   b) Find the time at which the ball hits the ground. **(3 marks)**

   c) Find the time at which the ball is at its greatest height and find this height. **(3 marks)**

5. A sealed cylindrical can of height h cm and radius r cm has a total surface area of 24π cm^2 and a volume of V cm^3.

   a) Write down an expression for the surface area and show that $h = \dfrac{12 - r^2}{r}$. **(3 marks)**

   b) Obtain an expression for V in terms of r and hence find the value of r which will maximise the volume. Find this volume, and verify that your answer is indeed a maximum and not a minimum. **(7 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **differentiation**

---

1. a) $3x^2 - 12x + 9 > 0$ M1

$(x-1)(x-3) > 0$ A1

$x < 1,\ x > 3$ M1A1

2. a) $y = \dfrac{3}{x^3} - \dfrac{1}{x}$ M1

$\dfrac{-9}{x^4} + \dfrac{1}{x^2} = 0$ M1

$x^2 = 9$ A1

$x = 3 \Rightarrow y = \dfrac{3-9}{27} = -\frac{2}{9}$ A1

$x = -3 \Rightarrow y = \dfrac{3-9}{-27} = \frac{2}{9}$ A1

$\dfrac{d^2y}{dx^2} = \dfrac{36}{x^5} - \dfrac{2}{x^3}$ M1

$\left(3, -\frac{2}{9}\right) \Rightarrow \dfrac{d^2y}{dx^2} = \frac{2}{27} \Rightarrow$ minimum A1

$\left(-3, \frac{2}{9}\right) \Rightarrow \dfrac{d^2y}{dx^2} = -\frac{2}{27} \Rightarrow$ maximum A1

b) $5x^4 - 45x^2 = 0 \Rightarrow 5x^2(x^2 - 9) = 0$ M1

$x = 0 \Rightarrow y = 0$ A1

$x = 3 \Rightarrow y = 243 - 405 = -162$ A1

$x = -3 \Rightarrow y = -243 + 405 = 162$ A1

$\dfrac{d^2y}{dx^2} = 20x^3 - 90x$ M1

$(0,0) \Rightarrow \dfrac{d^2y}{dx^2} = 0 \Rightarrow$ inconclusive M1

gradient negative either side of (0, 0)

point of inflexion A1

$x = 3 \Rightarrow \dfrac{d^2y}{dx^2} = 270 \Rightarrow$ minimum A1

$x = -3 \Rightarrow \dfrac{d^2y}{dx^2} = -270 \Rightarrow$ maximum A1

3. a) $6x - 3x^2$ M1

b) $3x(2-x) = 0$ M1

(0, 0) and (2, 4) M1A1

$\dfrac{d^2y}{dx^2} = 6 - 6x$ M1

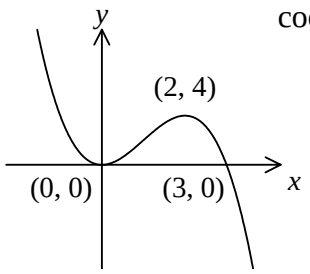
$(0,0) \Rightarrow \dfrac{d^2y}{dx^2} = 6 \Rightarrow$ minimum A1

$(2,4) \Rightarrow \dfrac{d^2y}{dx^2} = -6 \Rightarrow$ maximum A1

c) $3x(2-x) < 0 \Rightarrow x < 0,\ x > 2$ M1A1

d) coordinates B2

shape G1



4. a) 2 metres B1

b) $2 + 9t - 5t^2 = 0$ M1

$(5t+1)(t-2) = 0$ M1

$t = 2$ seconds A1

c) $9 - 10t = 0$ M1

$t = 0.9$ seconds, $h = 6.05$ metres A1A1

5. a) $2\pi r^2 + 2\pi rh = 24\pi$ M1

$r^2 + rh = 12$ M1

$h = \dfrac{12 - r^2}{r}$ A1

b) $V = \pi r^2\left(\dfrac{12 - r^2}{r}\right) = 12\pi r - \pi r^3$ M1A1

$12\pi - 3\pi r^2 = 0$ M1

$r = 2$ cm A1

$V = 24\pi - 8\pi = 16\pi$ cm^3 A1

$\dfrac{d^2V}{dr^2} = -6\pi r$ neg so maximum M1A1

A– LEVEL TOPIC REVIEW

unit C2 **integration**

---

1. Evaluate:

a) $\int_2^5 2x^2 + 1\,dx$ b) $\int_1^2 \frac{1}{x^2}\,dx$ c) $\int_1^4 \frac{x^2 + x^3}{\sqrt{x}}\,dx$ **(9 marks)**

2. a) Sketch on the same diagram the graphs of $y = x^2 + 2$ and $2x + y = 5$ for $0 \le x \le 3$.

(3 marks)

b) Find the coordinates of the point of intersection of the two graphs. **(3 marks)**

c) Use integration to calculate the area enclosed by $2x + y = 5$, $y = x^2 + 2$ and the *y*-axis.

(3 marks)

3. Use the trapezium rule with the number of trapezia indicated to find approximations to the following integrals.

a) $\int_1^7 \frac{1}{x}\,dx$, 6 trapezia b) $\int_0^2 2^x\,dx$, 4 trapezia **(6 marks)**

4. Find the areas enclosed by the following lines and curves. In each case draw a sketch to show the area concerned.

a) $y = x^2 - 5x + 6$ and the *x*-axis. **(5 marks)**

b) $y = x^2 - 3x$ and $y = x$. **(7 marks)**

5. a) Sketch the curve $y = x(x-1)(x-2)$. **(2 marks)**

b) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}$. **(5 marks)**

c) Show this tangent meets the curve again at (2, 0) and draw the tangent on your sketch.

(2 marks)

d) Find the area enclosed between the tangent, the curve and the *x*-axis. **(5 marks)**

A– LEVEL TOPIC REVIEW : ANSWERS

unit C2 **integration**

---

1. a) $\left[\frac{2}{3}x^3+x\right]_2^5$ M1

$\left(\frac{250}{3}+5\right)-\left(\frac{16}{3}+2\right)=81$ M1A1

b) $\left[-\frac{1}{x}\right]_1^2$ M1

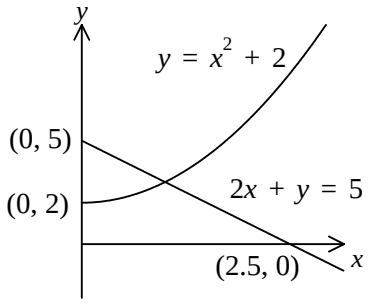
$-\frac{1}{2}--1=\frac{1}{2}$ A1

c) $\int_1^4 x^{\frac{3}{2}}+x^{\frac{5}{2}}\,dx$ A1

$\left[\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{7}x^{\frac{7}{2}}\right]_1^4$ M1

$\left(\frac{64}{5}+\frac{256}{7}\right)-\left(\frac{2}{5}+\frac{2}{7}\right)=48\frac{24}{35}$ M1A1

2. a) G3



b) $x^2+2=5-2x$ M1

$x^2+2x-3=0$

$(x+3)(x-1)=0 \Rightarrow (1,\ 3)$ M1A1

c) $\int_0^1 5-2x\,dx-\int_0^1 x^2+2\,dx$ M1

$\int_0^1 3-2x-x^2\,dx$ A1

$\left[3x-x^2-\frac{1}{3}x^3\right]_0^1$

$\left(3-1-\frac{1}{3}\right)-0=1\frac{2}{3}$ A1

3. a) $\frac{1}{2}\left(\frac{1}{1}+2\frac{1}{2}+2\frac{1}{3}+2\frac{1}{4}+2\frac{1}{5}+2\frac{1}{6}+\frac{1}{7}\right)$ M1A1

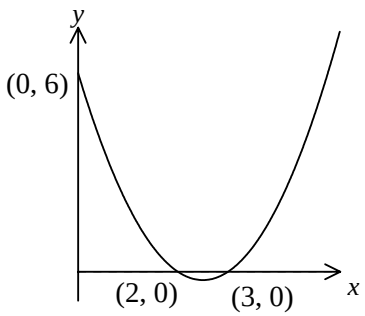
$2\frac{3}{140}$ or 2.0214… A1

b) $\frac{\frac{1}{2}}{2}\left(1+2\sqrt{2}+2\times2+2\times2\sqrt{2}+4\right)$ M1A1

$\frac{9}{4}+\frac{3}{2}\sqrt{2}$ or 4.3717… A1

4. a) $(x-2)(x-3)=0 \Rightarrow x=2,\ x=3$ M1

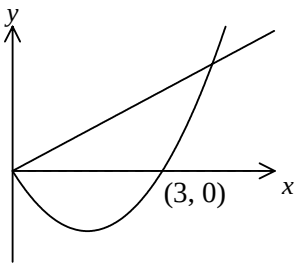
G2



$\left[\frac{1}{3}x^3-\frac{5}{2}x^2+6x\right]_2^3$ M1

$\left(9-\frac{45}{2}+18\right)-\left(\frac{8}{3}-10+12\right)=-\frac{1}{6}$ A1

b) G2



$x^2-3x=x$ M1

$x^2-4x=0 \Rightarrow x=0,\ x=4$ A1

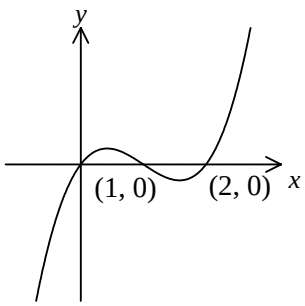
$\int_0^4 x\,dx-\int_0^4 x^2-3x\,dx$ M1

$\int_0^4 4x-x^2\,dx$

$\left[2x^2-\frac{1}{3}x^3\right]_0^4$ M1

$\left(32-\frac{64}{3}\right)-0=10\frac{2}{3}$ A1

6. a) G2



b) $y = x^3 - 3x^2 + 2x$ M1

$x = \frac{1}{2} \Rightarrow y = \frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} = \frac{3}{8}$ A1

$\frac{dy}{dx} = 3x^2 - 6x + 2$ M1

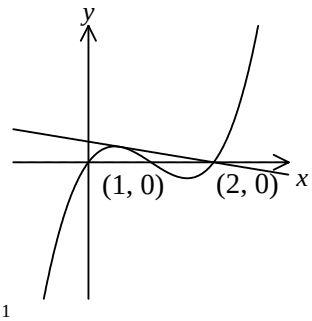
$\frac{dy}{dx} = 3 \times \frac{1}{4} - 6 \times \frac{1}{2} + 2 = -\frac{1}{4}$ A1

$y - \frac{3}{8} = -\frac{1}{4}\left(x - \frac{1}{2}\right)$

$y = -\frac{1}{4}x + \frac{1}{2}$ A1

c) $x = 2 \Rightarrow y = -\frac{1}{4} \times 2 + \frac{1}{2} = 0$ A1

G1



d) $\frac{1}{2}bh - \int_{\frac{1}{2}}^{1} x^3 - 3x^2 + 2x\,dx$ M1

$\frac{1}{2} \times \frac{3}{2} \times \frac{3}{8} = \frac{9}{32}$ A1

$\left[\frac{1}{4}x^4 - x^3 + x^2\right]_{\frac{1}{2}}^{1}$ M1

$\left(\frac{1}{4} - 1 + 1\right) - \left(\frac{1}{64} - \frac{1}{8} + \frac{1}{4}\right) = \frac{7}{64}$ A1

$\frac{9}{32} - \frac{7}{64} = \frac{11}{64}$ A1